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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION, FEBRUARY 2017 (2015 ADMISSION)

Course Code: MA 101 Course Name: CALCULUS

Max.Marks: 100

Duration: 3 Hours

PART A

(Answer all questions. Each question carries 3 marks)

1) Show that the series
$$\sum_{n=1}^{\alpha} \left(\frac{1}{2}\right)^n$$
 converges.

2) Classify the surface $z = (x - 1)^2 + (y + 2)^2 + 3$

3) Find the Maclaurin series for $\cos x$

4) Evaluate $\lim_{(x,y)\to(-1,2)} \frac{xy}{x^2 + y^2}$

- 5) Convert the cylindrical co-ordinate into rectangular co ordinate of $(4, \pi/3 3)$.
- 6) Find the slope of the surface $z = xy^2$ in the x direction at the point (2,3).
- 7) Find the directional derivative of $f = x^2 y y z^3 + z$ at (1,-2,0) in the direction of

$$\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$$

8) Find the unit normal to the surface xy + xz + yz = c at (-1,2,3)

9) Evaluate
$$\int_{1}^{a} \int_{1}^{b} x^2 y \, dx \, dy$$

10) Find the area of the region R enclosed by y = 1, y = 2, x = 0, x = y.

PART B

(Answer any 2 questions. Each question carries 7 marks)

11) Test the absolute convergence of $\sum_{n=1}^{\alpha} \frac{(-1)^n n^4}{4^n}$

12) Determine the Taylor's series expansion of f(x) = sin x at $x = \pi/2$.

13) Test the convergence of
$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5}$$
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(Answer any 2 questions. Each question carries 7 marks)

- 14) Find the equation of the paraboloid $z = x^2 + y^2$ in the cylindrical and spherical coordinates.
- 15) Find F(f(x),g(y),h(z)) if $F(x,y,z) = y e^{xyz}$, $f(x) = x^2$, g(y) = y + 1, $h(z) = 2z^2$
- 16) By converting into polar coordinate evaluate $\lim_{(x,y)\to(0,0)} \sqrt{x^2 + y^2} \ln\left(\left(x^2 + y^2\right)^2\right)$

(Answer any 2 questions. Each question carries 7 marks)

17) Find the local linear approximation L of f(x,y,z) = xyz at the point P(1,2,3). Compare the error in approximating f by L at the point Q(1.001, 2.002, 3.003) with the distance PQ. 18) Find the relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$

19) If f is a differentiable function of three variables and suppose that

$$w = f(x - y, y - z, z - x)$$
 Show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$

(Answer any 2 questions. Each question carries 7 marks)

- 20) Suppose that a particle moves along a curve in 3-space so that its position vector at time t is $r(t) = 4\cos \pi t i + 4\sin \pi t j + t k$. Find the distance travelled and the displacement of the particle during the time interval $1 \le t \le 5$
- 21) A particle is moving along the curve, $\vec{r} = (t^3 2t)\vec{i} + (t^2 4)\vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at t = 1. Also find the vector tangential and normal components of acceleration at t = 0.
- 22) Find the arc length of the parametric curve $x = 5\cos t$, $y = 5\sin t$, z = 2t; $0 \le t \le \pi$

(Answer any 2 questions. Each question carries 7 marks)

- 23) Evaluate the integral by converting into polar co ordinates $\int_{0}^{2\sqrt{4-x^{2}}} \int_{0}^{2\sqrt{4-x^{2}}} (x^{2} + y^{2}) \, dy \, dx$
- 24) Using triple integral to find the volume bounded by the cylinder

 $x^2 + y^2 = 4$ and the planes z=0 and y+z=3

25) Change the order of integration and evaluate $\int_{0}^{1} \int_{x}^{1} \frac{x}{x^{2} + y^{2}} dx dy$

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