$\qquad$ Name $\qquad$

## APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION, FEBRUARY 2017 (2015 ADMISSION)

Course Code: MA 101
Course Name: CALCULUS
Max.Marks : 100
Duration: 3 Hours

## PART A

## (Answer all questions. Each question carries $\mathbf{3}$ marks)

1) Show that the series $\sum_{n=1}^{\alpha}\left(\frac{1}{2}\right)^{n}$ converges.
2) Classify the surface $z=(x-1)^{2}+(y+2)^{2}+3$
3) Find the Maclaurin series for $\cos x$
4) Evaluate $\operatorname{Lt}_{(x, y) \rightarrow(-1,2)} \frac{x y}{x^{2}+y^{2}}$
5) Convert the cylindrical co-ordinate into rectangular co ordinate of (4, $\pi / 3-3)$.
6) Find the slope of the surface $z=x y^{2}$ in the $x$ direction at the point $(2,3)$.
7) Find the directional derivative of $f=x^{2} y-y z^{3}+z$ at $(1,-2,0)$ in the direction of

$$
\vec{a}=2 \vec{i}+\vec{j}+2 \vec{k}
$$

8) Find the unit normal to the surface $x y+x z+y z=c$ at $(-1,2,3)$
9) Evaluate $\int_{1}^{a} \int_{1}^{b} x^{2} y d x d y$
10) Find the area of the region R enclosed by $y=1, y=2, x=0, x=y$.

## PART B

## (Answer any 2 questions. Each question carries 7 marks)

11) Test the absolute convergence of $\sum_{n=1}^{\alpha} \frac{(-1)^{n} n^{4}}{4^{n}}$
12) Determine the Taylor's series expansion of $\mathrm{f}(\mathrm{x})=\sin x$ at $\mathrm{x}=\pi / 2$.
13) Test the convergence of $\frac{1}{1.2 .3}+\frac{3}{2 \cdot 3 \cdot 4}+\frac{5}{3.4 .5} \ldots \ldots \ldots$

## (Answer any 2 questions. Each question carries 7 marks)

14) Find the equation of the paraboloid $z=x^{2}+y^{2}$ in the cylindrical and spherical coordinates.
15) Find $F(f(x), g(y), h(z))$ if $F(x, y, z)=y e^{x y z}, f(x)=x^{2}, g(y)=y+1, h(z)=2 z^{2}$
16) By converting into polar coordinate evaluate $\underset{(x, y) \rightarrow(0,0)}{\operatorname{Lt}} \sqrt{x^{2}+y^{2}} \ln \left(\left(x^{2}+y^{2}\right)^{2}\right)$

## (Answer any 2 questions. Each question carries 7 marks)

17) Find the local linear approximation L of $f(x, y, z)=x y z$ at the point $\mathrm{P}(1,2,3)$. Compare the error in approximating $f$ by $L$ at the point $Q(1.001,2.002,3.003)$ with the distance $P Q$.
18) Find the relative extrema of $f(x, y)=3 x^{2}-2 x y+y^{2}-8 y$
19) If f is a differentiable function of three variables and suppose that

$$
w=f(x-y, y-z, z-x) \quad \text { Show that } \frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0
$$

## (Answer any 2 questions. Each question carries 7 marks)

20) Suppose that a particle moves along a curve in 3-space so that its position vector at time $t$ is $r(t)=4 \cos \pi t i+4 \sin \pi t j+t k$. Find the distance travelled and the displacement of the particle during the time interval $1 \leq t \leq 5$
21) A particle is moving along the curve, $\vec{r}=\left(t^{3}-2 t\right) \vec{i}+\left(t^{2}-4\right) \vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t=1$. Also find the vector tangential and normal components of acceleration at $\mathrm{t}=0$.
22) Find the arc length of the parametric curve $\quad x=5 \cos t, y=5 \sin t, z=2 t ; 0 \leq t \leq \pi$

## (Answer any 2 questions. Each question carries 7 marks)

23) Evaluate the integral by converting into polar co ordinates $\int_{0}^{2 \sqrt{4-x^{2}}}\left(x_{0}^{2}+y^{2}\right) d y d x$
24) Using triple integral to find the volume bounded by the cylinder

$$
x^{2}+y^{2}=4 \text { and the planes } z=0 \text { and } y+z=3
$$

25) Change the order of integration and evaluate $\int_{0}^{1} \int_{x}^{1} \frac{x}{x^{2}+y^{2}} d x d y$
